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One might say that physicists study the symmetry of nature, while mathematicians study the nature of symmetry.

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@GWOMaths Observing symmetry in nature, such as noting the similarity between the symmetries of a snowflake and a hexagon, is readily comprehensible. What does it mean then to study "the nature of symmetry"?

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@GWOMaths Mathematicians define a "group", G , as a set of elements $\{a, b, c, \dots\}$ with a binary operation \blacksquare and a distinguished element e (the identity of G) satisfying these specific properties:

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@GWOMaths For all a, b, c in G

- 1) Closure: $a \blacksquare b$ is in G .
- 2) Associativity: $(a \blacksquare b) \blacksquare c = a \blacksquare (b \blacksquare c)$
- 3) Identity: $e \blacksquare a = a \blacksquare e = a$.
- 4) Inverse: There exists an element a^* such that $a^* \blacksquare a = a \blacksquare a^* = e$.

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@GWOMaths Just as functions on the integers, rationals, or reals are defined as mappings, mathematicians define a *group morphism* μ as a mapping from one group to another that preserves the group structure:

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@GWOMaths For groups $G = [\{a, b, c, \dots\}, \blacksquare, e]$ and $H = [\{\alpha, \beta, \gamma, \dots\}, \blacksquare, \epsilon]$

then $\mu: G \rightarrow H$ is a "group morphism" if for all elements of G :

$\mu(a \blacksquare b) = \mu(a) \blacksquare \mu(b)$. Note that for all a :

$\mu(a) \blacksquare \epsilon = \mu(a) = \mu(a \blacksquare e) = \mu(a) \blacksquare \mu(e)$

and hence $\mu(e) = \epsilon$; Similarly it can be shown that

$$\mu(a^*) = \mu(a)^*.$$

@GWOMaths Thus the definition of such a *group morphism* preserves the group structure. When such a *morphism* is both *onto* (ie every element of H is mapped to by one or more elements of G) and *one-to-one* (only one element of G maps to each element in H) it is termed an *isomorphism*.

@GWOMaths For mathematical purposes, when there exists an *isomorphism* between two groups $G = \{a, b, c, \dots, \mathbf{1}, e\}$ and $H = \{\alpha, \beta, \gamma, \dots, \mathbf{1}, \varepsilon\}$ then G and H are termed *isomorphic*, or *the same up to isomorphism*.

@GWOMaths Now all the finite groups can be classified in terms of various internal structures, first collecting those which are *the same up to isomorphism* and then collecting families with similar internal structure.

@GWOMaths When all the families of groups - Cyclic, Alternating, and assorted Lie Group Types - have been defined, there are remaining 26 groups that don't fit anywhere: the *sporadic groups*.

@GWOMaths Of these 26 *sporadic groups*, two stand out from the others in terms of their size:

- the *Baby Monster*, *B*, of size

$$2^{\mathbf{1}} \cdot 3^{13} \cdot 5^{\mathbf{1}} \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 31 \cdot 47; \text{ and}$$

@GWOMaths - the *(Fischer–Griess) Monster*, *M*, of size

$$2^{\mathbf{15}} \cdot 3^2 \cdot 5^{\mathbf{1}} \cdot 7^{\mathbf{1}} \cdot 11^2 \cdot 13^3 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71.$$

Counting up the number of distinct primes in that last number gives us 15.

@GWOMaths Therefore today's answer is that:

The number of distinct prime factors
in the size, n, of the *Monster Group* M
is

15.

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